Electroweak Symmetry Breaking as a Consequence of Compact Dimensions

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Abstract

It has been shown recently that the Higgs doublet may be composite, with the left-handed top-bottom doublet and a new right-handed anti-quark as constituents bound by some four-quark operators with non-perturbative coefficients. I show that these operators are naturally induced if there are extra spatial dimensions with a compactification scale in the multi-TeV range. The Higgs compositeness is due mainly to the Kaluza-Klein modes of the gluons, while flavor symmetry breaking may be provided by various fields propagating in the compact dimensions. I comment briefly on the embedding of this scenario in string theory.

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Recently it has been shown that a bound state formed of the left-handed top-bottom quark doublet and a new right-handed anti-quark can play viably the role of the Standard Model (SM) Higgs doublet [1, 2]. The only ingredients necessary in this framework are some four-quark interactions, suppressed by a multi-TeV scale M_1. One is then led to ask what is the origin of these non-renormalizable interactions. The traditional answer is that they are produced by the exchange of some heavy gauge bosons associated with the breaking of a larger gauge group down to the QCD group. The simplest choice of this sort is the topcolor [3] scheme: $SU(N_c)_1 \times SU(N_c)_2 \rightarrow SU(N_c)_C$, where $N_c = 3$, the third generation quarks are charged under $SU(N_c)_1$, and the first two generations are charged under $SU(N_c)_2$. This choice has several nice features. For example, the asymptotic freedom of the topcolor gauge group allows the heavy gauge bosons to be strongly coupled at the scale M_1 without problems with a Landau pole. Also, it is technically convenient because it allows the use of the large-N_c expansion.

The drawback is that one has to include additional structures in order to spontaneously break topcolor. Moreover, this embedding of the $SU(N_c)_C$ color group may be seen as artificial. The embedding can be improved by allowing all quarks to transform only under $SU(N_c)_1$ [4] which is then embedded in a gauged flavor or family symmetry [5, 6]. However, such an extension requires a more complicated topcolor breaking sector.

It is therefore legitimate to ask whether one can avoid the extension of the gauge group while inducing the desired four-quark operators and retaining the nice features of topcolor ? In this letter I point out that the required four-quark operators are naturally induced if the SM gauge fields propagate not only in the usual four-dimensional Minkowski space-time but also in a compact space.

The first step in showing this is to recall that the existence of the extra dimensions is manifested within the 4-dimensional space-time through a tower of Kaluza-Klein (KK) modes associated with each of the fields propagating in the compact manifold (see *e.g.*, [7, 8]). The KK modes of a massless gauge boson have masses between $M_1 = R_{\mathrm{max}}^{-1}$, where R_{max} is the largest compactification radius, and the fundamental scale associated with quantum gravity, M_*. Assuming for simplicity that the fermions propagate at the fixed points of an orbifold, so that they have only zero-modes, it follows that the couplings of the KK modes of the gauge bosons to the quarks and leptons are identical (up to an overall normalization) with those of the SM gauge bosons. Thus, the KK excitations of

the gluons give rise in the low energy theory to flavor universal four-quark operators:

$$\mathcal{L}_{\text{eff}}^c = -\frac{cg_s^2(M_1)}{2M_1^2} \left(\sum_q \overline{q}\gamma_\mu T^a q \right)^2 \,, \tag{1}$$

where q are all the quarks, T^a are the $SU(N_c)_{\text{C}}$ generators, and g_s is the QCD gauge coupling. The dimensionless coefficient $c > 0$ sums the contributions of all gluonic KK modes, so that it depends on the number of extra dimensions and on the compactification radii. Higher dimensional operators are also induced in the low energy theory, but the contribution from a gluonic KK mode of mass M_n is suppressed compared to the contribution to c by powers of M_n/M_1. Therefore, the effects of the higher dimensional operators may be neglected. By contrast, in topcolor models the higher dimensional quark operators induced by the heavy gauge bosons are usually ignored for convenience; this procedure may be physically reasonable but so far has not been mathematically motivated.

The contact interaction $\mathcal{L}_{\text{eff}}^c$ is attractive in the scalar channel, so that spinless $\overline{q}_L q_R$ bound states form. Their properties can be studied using an effective potential formalism. In the large-N_c limit only the left-right current-current part of $\mathcal{L}_{\text{eff}}^c$ contributes to the effective potential for the composite scalars. Note that a Fierz transformation of this current-current interaction gives the well known Nambu–Jona-Lasinio (NJL) interaction. Hence, the large-N_c limit and the NJL model are equivalent approximations of the gluonic KK dynamics. For c larger than a critical value, the composite scalars acquire electroweak asymmetric VEVs [9]. Ignoring the renormalization group evolution of c above the scale M_1, one can find the critical value to leading order in $1/N_c$:

$$c_{\text{crit}} = \frac{2\pi}{N_c\alpha_s(M_1)} \,, \tag{2}$$

where $\alpha_s = g_s^2/4\pi$. The important feature here is that the chiral phase transition is second order as c is varied. This property is expected to remain true beyond the large-N_c approximation [10]. In the absence of excessive fine-tuning, $i.e.$ if c is not very close to c_{crit}, the electroweak scale of 246 GeV indicates that M_1 is in the multi-TeV range, or smaller.

The next step is to decide whether the four-quark operators induced by the gluonic KK modes can be super-critical. In principle the answer is positive, because the higher-dimensional gauge coupling is dimensionfull such that the strength of the gauge interactions increases rapidly above M_1 [11].

It would be useful to find the dimensionality and topology of the compact manifold necessary for electroweak symmetry breaking (EWSB). For simplicity, consider a

δ-dimensional torus or orbifold with radii R_l, $l = 1, ..., \delta$. The spectrum of the KK excitations of a massless field is given by

$$M^2_{n_1,...,n_\delta} = \sum_{l=1}^{\delta} \frac{n_l^2}{R_l^2} \, , \tag{3}$$

where n_l are integers (KK excitation numbers). In what follows, the KK mass levels will be denoted by M_n with n integer ($1 \leq n \leq n_{\max}$ where $M_{n_{\max}} \approx M_*$), and their degeneracy by D_n. To compute the coefficient of the four-quark operator, one would need to integrate out the $D_{n_{\max}}$ modes of mass $M_{n_{\max}}$, then to use the renormalization group evolution for c from $M_{n_{\max}}$ down to $M_{n_{\max}-1}$, and to repeat these steps for each KK mass threshold until the lightest states are integrated out. This would give the coefficient c at the scale M_1 as a function of M_*/R_l and $g_s(M_1)$.

Fortunately it is not necessary to perform this computation in order to show that there are compact manifolds which induce EWSB. Moreover, a continuity argument shows that c may be tuned close to the critical value. To see this, note that the contribution from any gluonic KK state to c is positive, so that truncating the tower of states at some $n_{\mathrm{tr}} < n_{\max}$ gives $c(n_{\mathrm{tr}}) < c(n_{\max}) = c$. Furthermore, the running of c may be ignored if $M_{n_{\mathrm{tr}}}$ is sufficiently close to M_1. For example, if the δ extra dimensions have the same compactification radius, R, then the truncation of the KK tower at $M_{n_{\mathrm{tr}}} = 2/R$ yields

$$c(n_{\mathrm{tr}} = 4) = \sum_{n=1}^{4} \frac{D_n}{n} \, , \tag{4}$$

where the degeneracy is given for $n \leq 7$ by

$$D_n = 2^n \, \delta! \left[\frac{\theta(\delta - n)}{n! \, (\delta - n)!} + \frac{\theta(n - 4)\theta(\delta - n + 3)}{8 \, (n - 4)! \, (\delta - n + 3)!} \right] \, , \tag{5}$$

with the step function $\theta(x \geq 0) = 1$. For $\delta = 4$, and M_1 in the multi-TeV range [where $\alpha_s(M_1) \approx 0.08 - 0.06$, corresponding to $c_{\mathrm{crit}} \approx 26 - 35$],

$$c > c(4) \approx 35.7 > c_{\mathrm{crit}} \, , \tag{6}$$

which shows that four extra dimensions are sufficient for the composite Higgs doublets to acquire VEVs. As a corollary, the SM is not viable if there are four or more extra dimensions with a compactification scale below $M_*/2$, because the quarks would acquire dynamical masses of the order of the compactification scale.

Consider now the case $\delta = 1$. Ignoring the running of c, the sum over the contributions from all KK states yields

$$c = \sum_{n_1=1}^{n_{\max}} \frac{2}{n_1^2} < \frac{\pi^2}{3} < c_{\mathrm{crit}} \ . \tag{7}$$

Even if the contributions from the running of c happen to be positive, it seems unlikely that they amount to the factor of order 10 required to drive c over the critical value. Thus, if there is only one compact dimension, it is fair to expect that the electroweak symmetry remains unbroken.

Coming back to the super-critical $\delta = 4$ case, one can imagine decreasing continuously three of the four radii while keeping M_* fixed. When these radii reach the value $1/M_*$, the case $\delta = 1$ is recovered. As a result, the coefficient c of the four-quark operator (1) decreases continuously (ignoring the fact that the number of gluonic KK modes below M_*, N_{KK}, is finite) from the super-critical case, $R_l = R$, $l = 1, ..., 4$, down to the sub-critical case, $R_1 = R$, $R_2 = R_3 = R_4 = 1/M_*$. In the large N_c limit, this decrease in the strength of the four-quark coupling is associated with a continuous change of the mass-squared of the composite $\overline{q}_L q_R$ scalars, from a negative value in the super-critical case to a positive value in the sub-critical case. Assuming that the results obtained in the large N_c limit are valid for $N_c = 3$, it follows that the decrease of some of the radii for fixed M_* induces a second order transition from the phase in which the electroweak symmetry is broken to the unbroken phase. The boundary of the phase transition corresponds to a set of manifolds with radii $R_1, ..., R_\delta$, where $2 \leq \delta \leq 4$. A more precise estimate of c might change the lower bound on δ. On the other hand, the above arguments show that there are 4-dimensional manifolds, with a hierarchy of compactification radii [at least one radius has to be shorter than $1/(2M_*)$], which yield c close to the critical value. This result opens up the possibility of constructing realistic models of Higgs compositeness based on compact dimensions.

Although the gluonic KK modes may induce EWSB, they do not provide flavor symmetry breaking. For super-critical c, $\mathcal{L}_{\mathrm{eff}}^c$ would produce an $SU(N_f)$ symmetric condensate and an $SU(N_f)$ adjoint of Nambu–Goldstone bosons (N_f is the number of quark flavors). All the quarks would acquire the same dynamical mass, related to the electroweak scale. It is thus necessary to identify a source of flavor symmetry breaking. Also, as explained in ref. [1, 2], at least one new quark, χ, should be introduced such that a $\overline{t}_L \chi_R$ dynamical mass of order 0.5 TeV is induced, leading to the observed W and Z masses.

The KK excitations of the hypercharge gauge boson give rise to four-fermion operators

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which are attractive for the up-type quarks and repulsive for the down-type quarks:

$$\mathcal{L}_{\text{eff}}^Y = -\frac{cg'^2}{M_1^2}\left(\frac{1}{3}\overline{\psi}_L^i\gamma_\mu\psi_L^i + \frac{4}{3}\overline{u}_R^i\gamma_\mu u_R^i - \frac{2}{3}\overline{d}_R^i\gamma_\mu d_R^i + \frac{4}{3}\overline{\chi}\gamma^\mu\chi\right)^2 , \tag{8}$$

where $i = 1, 2, 3$ is a generational index, $\psi_L^i = (u^i, d^i)_L$, and the lepton currents are not shown for simplicity. The vector-like quark χ transforms under the SM gauge group in the same representation as u_R^i. If the gluonic KK modes yield c within about 10% of its critical value, then the combination $\mathcal{L}_{\text{eff}}^c + \mathcal{L}_{\text{eff}}^Y$ induces VEVs only for the Higgs fields made up of the u, c, t and χ quarks.

At this stage it is necessary to introduce inter-generational flavor symmetry breaking. It is convenient to parametrize it using four-fermion operators of the following type:

$$\frac{\eta_{AB}}{M_1^2}\left(\overline{A}_L B_R\right)\left(\overline{B}_R A_L\right) , \tag{9}$$

with the notation $A_L = \psi_L^i, \chi_L$ and $B_R = u_R^k, d_R^i$, where $k = 1, ..., 4$, and $u_R^4 \equiv \chi_R$. Unlike the four-quark operators induced by the gluonic KK modes, which are strongly coupled and give rise to deeply bound states, the operators (9) can be treated perturbatively because the coefficients η_{AB} do not need to be larger than $\mathcal{O}(1)$.

The origin of these operators can be found in different scenarios for the physics at scales above M_1. For example, they can be produced at the fundamental scale M_*, where the most general gauge invariant operators are likely to be present. Note that the 't Hooft coupling $\alpha_s N_{KK}$ becomes non-perturbative not far above M_1, at a scale which is likely to be M_*. Such a low M_* may occur in the truly strong coupling regime of string theory [12], as well as when there are large dimensions inaccessible to the SM fields [13]. Predicting the coefficients of the flavor non-universal operators may require a complete theory that includes quantum gravitational effects [this also applies to the flavor violation in eq. (11) below]. Another possibility is that the flavor breaking operators are generated by various fields propagating in the large extra dimensions [11, 14]. Also, if the position of the quarks in the extra dimensions is flavor-dependent [15], then even flavor-universal interactions at the scale M_* would lead to non-universality in eq. (9).

If one of the $\eta_{\psi^i u^k}$ coefficients, chosen by convention to be $\eta_{\psi^3\chi}$, is larger than the other eleven, then the condition

$$\eta_{\psi^3\chi} > \frac{2\pi^2}{N_c} - c\left(g_s^2 + \frac{8}{9N_c}g'^2\right) > \eta_{\psi^i u^k} \tag{10}$$

implies that only the $\overline{\chi}_R\psi_L^3$ Higgs doublet has a negative squared mass, resulting in the hierarchy between the top quark and the others.

In order to accommodate the observed masses of the W, Z and t, a few other conditions must be satisfied [1, 2]. First, the gauge invariant mass term $\mu_{\chi\chi}\overline{\chi}_L\chi_R$ has to be included, so that tadpole terms for the $\overline{\chi}_L\chi_R$ scalar are induced in the effective potential. For this purpose, the $\mu_{\chi\chi}$ mass parameter can be significantly smaller than M_1. Such a small mass may arise naturally, for example from the VEV of a gauge singlet scalar which propagates in some compact dimensions which are inaccessible to χ (a similar mechanism is used in ref. [17] to produce neutrino masses). Generically, the $\mu_{\chi u^i}\overline{\chi}_L u_R^i$ mass terms are also present. A second condition for realizing the top condensation seesaw mechanism [1, 2] is to have the squared masses of the $\overline{\chi}_R\chi_L$ and $\overline{t}_R\chi_L$ scalars larger than the squared mass of the $\overline{\chi}_R\psi_L^3$ Higgs doublet, or equivalently: $\eta_{\psi^3\chi} > \eta_{\chi t}, \eta_{\chi\chi}$. These conditions ensure that the minimum of the effective potential for composite scalars corresponds to dynamical fermion masses only for the t and χ. The $\overline{t}_L\chi_R$ mass mixing, responsible for EWSB, has to be of order 0.5 TeV, while the top mass measurements and the constraint on custodial symmetry violation require the fermion mass eigenvalues to be $m_t \approx 175$ GeV and $m_\chi \gtrsim 3$ TeV. In the absence of excessive fine-tuning, $m_\chi \sim \mathcal{O}(5)$ TeV corresponds to a compactification scale M_1 of order 10 – 50 TeV.

The gluonic KK excitations are responsible for the existence of N_f^2 composite complex scalars, each of the left-handed quark flavors binding to each of the right-handed ones. Their mass degeneracy is lifted by the flavor non-universal four-fermion operators (9), but generically most of the physical states have masses of order m_χ [2]. However, the composite scalar spectrum includes the longitudinal W and Z, and a neutral Higgs boson which is always lighter than 1 TeV, and may be as light as $\mathcal{O}(100)$ GeV if the vacuum is close to the boundary of a second order phase transition [2]. In the decoupling limit, where $m_\chi \to \infty$, the low energy theory is precisely the SM, with the possible addition of other composite states which may be light due to the vicinity of a phase transition.

So far, the electroweak symmetry is broken correctly, m_t is accommodated, and the χ quark has a mass of at least a few TeV. It remains to produce the masses and mixings of the other quarks and leptons. For this reason, consider the following four-fermion operators, assumed to be produced by physics above the compactification scale:

$$\frac{1}{M_1^2}\left(\overline{\chi}_R\psi_L^3\right)\left[\xi_{\psi^j u^k}\left(\overline{\psi}_L^j u_R^k\right) + \xi_{\psi^j d^k}\left(\overline{\psi}_L^j i\sigma_2 d_R^k\right) + \xi_{l^j\nu^k}\left(\overline{l}_L^j \nu_R^k\right) + \xi_{l^j e^k}\left(\overline{l}_L^j i\sigma_2 e_R^k\right)\right] , \quad (11)$$

where l_L^j, ν_R^j and e_R^j are the SM lepton fields. These operators lead through the renormalization group evolution to SM Yukawa couplings (proportional with the dimensionless coefficients ξ) of the $\overline{\chi}_R\psi_L^3$ composite Higgs doublet to the fermions. Another effect of the

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operators (11) is to mix the $\overline{\chi}_R \psi_L^3$ Higgs doublet with the other $\overline{q}_R^i \psi_L^j$ composite scalars.

The flavor breaking operators (9) and (11) do not produce large flavor-changing neutral current (FCNC) effects beyond those in the SM. It remains to be shown however that these operators can be induced by some high energy physics without being accompanied by flavor-changing operators with large neutral-current effects. Another contribution to the FCNCs comes from the composite scalars, which have flavor breaking couplings to the quark mass-eigenstates. These are suppressed by Kobayashi-Maskawa elements, and they are not worrisome if the scalars are sufficiently heavy. It is beyond the scope of this letter to derive the constraints from FCNCs, but it is worth reiterating that the extra dimensions open up new ways of dealing with flavor. In addition, the low energy theory is flexible: for example the four-quark operators may be kept flavor universal by allowing flavor violation to arise from the mass terms of an extended vector-like quark sector [16, 5].

Placing the fermions at orbifold fixed points is convenient because all the gluonic KK modes contribute in this case at tree level to the four-quark operators. However, if at least one right-handed up-type quark propagates in compact dimensions, its KK modes could play the role of the vector-like quark, χ, required to account for the bulk of EWSB. This possibility requires further study because the couplings of the quark and gluonic KK modes are restricted by momentum conservation in the compact dimensions.

It is instructive to see how the scenario presented here may arise from string theory or M theory. The most convenient setting for studying large extra dimensions is within Type I string theory [8, 12, 18]. The closed string sector gives rise to the graviton and other neutral modes which propagate in the bulk of the 9+1 dimensional space-time. The open string sector gives rise to the gauge fields and the charged matter, which are restricted to propagate on a D9-brane or a D5-brane. Using T-duality transformations, one may obtain Type I′ string theories containing Dp-branes with $p \leq 9$. A $(\delta + 3)$-brane, with $2 \leq \delta \leq 4$, is necessary for containing the 3+1 dimensional flat space-time plus the δ compact dimensions that lead to a composite Higgs sector. Some of the δ compactification scales are expected to be of order $M_1 \sim 10 - 50$ TeV, while other must be slightly higher, in order to allow the coefficients of the four-quark operators to be close to criticality.

Due to the presence of the KK modes of the SM gauge bosons, the gauge couplings tend to unify at a scale higher by at most one order of magnitude than the compactification scale [8]. The running of the gauge couplings in the model discussed here is different than in the supersymmetric SM because below M_1 there are no superpartners, and there is a potentially complicated composite Higgs sector. Nevertheless, the gauge coupling

unification may be realized in various ways, due to the possible existence of additional states below or above M_1. Alternately, the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge groups may come from different types of branes, so that the gauge couplings need not unify precisely.

It is natural to identify the unification scale with the string scale, so that $M_s \sim \mathcal{O}(100)$ TeV, which also corresponds to the scale where $\alpha_s N_{KK} \sim 1$. This prediction is a consequence of the fit to the electroweak scale and top-quark mass within the top seesaw theory of Higgs compositeness. The low M_s is associated with large extra dimensions accessible only to gravity [13]. If these are identified with the $6 - \delta$ dimensions orthogonal to the $(\delta + 3)$-brane, then their compactification radius is given by

$$ r' \sim \frac{1}{M_s} \left[\alpha_s(M_s) \left(M_s^\delta R_1 ... R_\delta \right)^{1/2} \frac{M_{\mathrm{Planck}}}{M_s} \right]^{\frac{2}{(6-\delta)}} \ . \tag{12} $$

Using the value $\alpha_s(M_s) \sim 1/50$ for the gauge coupling at the string scale, $M_{\mathrm{Planck}} \sim 10^{19}$ GeV, and $R_1 ... R_\delta \sim (30 \text{ TeV})^\delta$ gives $r' \sim 10^{-6}$ cm for $\delta = 4$, and $r' \sim 10^{-10}$ cm for $\delta = 3$. Due to the low M_s, there may be stringy effects that change the physics at the scale M_1, including the coefficients of operators induced by gluonic KK modes. Whether the picture described here changes qualitatively remains an open question, together with other issues in the context of D-branes, such as how to construct stable non-supersymmetric brane configurations, or what branes correspond to the three generations of chiral fermions [19].

In conclusion, the top condensation seesaw mechanism is a compelling scenario for EWSB. It is remarkable that viable models involving this mechanism do not require an extension of the SM gauge group, provided there are a few compact dimensions accessible to the gluons. If the future collider experiments will probe the composite Higgs sector, we will be able to test the existence of the KK modes of the gluons, and consequently the structure of the space-time.

References

[1] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. **81** (1998) 2634 .

[2] R. S. Chivukula, B. A. Dobrescu, H. Georgi, and C. T. Hill, Phys. Rev. **D59** (1999) 075003.

[3] C. T. Hill, Phys. Lett. **B266** (1991) 419.

[4] R. S. Chivukula, A. G. Cohen, and E. H. Simmons, Phys. Lett. **B380** (1996) 92; E. H. Simmons, Phys. Rev. **D 55** (1997) 1678; M. B. Popovic and E. H. Simmons, Phys. Rev. **D58** (1998) 095007.

[5] G. Burdman and N. Evans, Phys. Rev. **D59** (1999) 115005.

[6] M. Lindner and G. Triantaphyllou, Phys. Lett. **B430** (1998) 303; G. Triantaphyllou, hep-ph/9811250.

[7] I. Antoniadis, Phys. Lett. **B246** (1990) 377; V. A. Kostelecky and S. Samuel, Phys. Lett. **B270** (1991) 21: I. Antoniadis and K. Benakli, Phys. Lett. **B326** (1994) 69; I. Antoniadis, K. Benakli and M. Quiros, Phys. Lett. **B331** (1994) 313.

[8] K. R. Dienes, E. Dudas, and T. Ghergheta, Phys. Lett. **B436** (1998) 55; Nucl. Phys. **B537** (1999) 47.

[9] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. **D41**, 1647 (1990).

[10] R. S. Chivukula and H. Georgi, Phys. Rev. **D58**, 075004 (1998).

[11] N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353.

[12] J. Lykken, Phys. Rev. **D54**, 3693 (1996).

[13] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429**, 263 (1998).

[14] Z. Berezhiani and G. Dvali, hep-ph/9811378.

[15] N. Arkani-Hamed and M. Schmaltz, hep-ph/9903417.

[16] E. H. Simmons, Nucl. Phys. **B324**, 315 (1989).

[17] K. R. Dienes, E. Dudas, T. Ghergheta, hep-ph/9811428; N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, hep-ph/9811448.

[18] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436**, 257 (1998); G Shiu and S.-H. H. Tye, Phys. Rev. **D58**, 106007 (1998); Z. Kakushadze and S.-H. H. Tye, hep-th/9809147.

[19] J. Lykken, E. Poppitz, S. P. Trivedi, Nucl. Phys. **B543** (1999) 105.